

Long beating wavelength in the Schwarz-Hora effect

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The quantum-mechanical interpretation of the long-wavelength spatial beating of the light intensity in the Schwarz-Hora effect is discussed. A more accurate expression for the spatial period has been obtained, taking into account the mode structure of the laser field within the dielectric film. It is shown that the discrepancy of more than 10% between the experimental and theoretical results for the spatial period cannot be reduced by using the existing models. More detailed experimental information is necessary to clear up the situation.

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In 1969, Schwarz and Hora [1] reported the results of an experiment in which a 50-keV beam of electrons passed through a thin crystalline film of SiO_2 , Al_2O_3 , or SrF_2 irradiated with laser light. Electrons produced the usual electron-diffraction pattern at a fluorescent target. However, the diffraction pattern was also observed at a nonfluorescent target [1–3] (the Schwarz-Hora effect). In this case the pattern was roughly of the same color as the laser light. The effect was absent if the electrical vector of the polarized laser light was parallel to the film surfaces. When changing the distance between the thin crystalline film and the target, a periodic change in the light intensity was observed with spatial period of the order of centimeters [2]. The Schwarz-Hora effect was discussed extensively in the literature in the early 1970s. The latest review can be found in Ref. [4].

The reported quantitative results [1–4] were obtained for the films of about 1000 Å thickness. The films were illuminated by a 10^7-W/cm^2 argon ion laser irradiation ($\lambda_p = 4880$ Å) perpendicular to the electron beam of about 0.4 μA current. These values will be used below for numerical estimates.

The quantum-mechanical treatment of the problem was made in the one-electron [2,4–8] and many-electron [9–11] approximations. One problem unresolved up to now is connected with the theoretical interpretation of the relatively high intensity of the Schwarz-Hora radiation (at least of the order of 10^{-10} W). The calculated radiated power turns out to be at least 10^3 times smaller than the observed power [4,7,9–12]. The other problem is connected with the strong dependence of the Schwarz-Hora radiation intensity on the laser light polarization [2,4,9]. An explanation of this dependence is absent too. In the following discussion, we do not consider these two problems.

In this Brief Report we consider only the more transparent problem connected with the interpretation of the

long-wavelength spatial modulation of the Schwarz-Hora radiation [2,4–7,9–11,13,14]. The one-particle and many-particle models lead to the same expression for the long beating wavelength. At first sight, there is even a good quantitative agreement with experiment [14]. However, as we shall see below, this agreement is accidental. Moreover, there is the discrepancy of more than 10% that cannot be reduced on the basis of the existing quantum theories.

Let the z axis be directed along the incident electron beam. The laser beam is along the x axis. The electrical vector of the laser light is in the z direction. Electrons pass through the dielectric slab restricted by the planes $z = -d$ and $z = 0$. We consider without loss of generalization only the central outgoing electron beam (zeroth-order diffraction).

Usually the following assumptions are used: An electron interacts with the light wave only within the slab; it interacts within the slab only with the light wave; the spin effects can be neglected. In the simplest case the light field within the slab and incident electrons are represented by plane waves.

Using these assumptions, consider the origin of the long-wavelength spatial modulation in the one-electron quantum theory. The solution of the Klein-Gordon equation to first order in the light field (see, for example, Refs. [5,7]) gives the following expression for the electron probability density for $z > 0$:

$$\rho(x, z, t) = \rho_0 \left\{ 1 - \beta \sin \left[\frac{z}{2\hbar} (2p_0 - p_{1z} - p_{-1z}) \right] \times \sin \left(\frac{\pi d}{2d_0} \right) \cos \left[kx - \omega t + \frac{z}{2\hbar} (p_{1z} - p_{-1z}) \right] \right\}. \quad (1)$$

Here ρ_0 is the probability density for the initial incident electron beam and ω and k denote the circular frequency and the wave number of the light wave inside the slab. The parameter β is proportional to the amplitude of the laser field and d_0 is the smallest optimum value of the slab thickness. For the conditions of the Schwarz experiments, these parameters are $\beta = 0.35$ (for α -quartz) and $d_0 = 1007$ Å. The z components of the momentum p_{nz} are determined for free electrons of energy E_n and momentum \mathbf{p}_n from the relativistic relationship

$$E_n^2 = m^2 c^4 + \mathbf{p}_n^2 c^2, \quad (2)$$

$$E_n = E_0 + n\hbar\omega, \quad p_{nx} = n\hbar k, \quad n = 0, \pm 1.$$

Here m is the electron mass.

The probability that an electron absorbs or emits a photon inside the dielectric slab is a periodic function of the slab thickness. This is indicated by the second sine term in Eq. (1). The experimental data on such dependence of the Schwarz-Hora radiation are absent in the literature. The cosine term represents the optical modulation of the electron beam. The first sine term in Eq. (1) is a function of the distance z between the slab and the target and represents the stationary modulation of the electron probability density. On equating the phase of this sine to $2\pi z/\lambda_b$, we obtain the expression for the spatial beating wavelength (the same expression is obtained in the many-electron treatment [9–11])

$$\lambda_b = \frac{4\pi\hbar}{2p_0 - p_{1z} - p_{-1z}}. \quad (3)$$

Taking into account Eq. (2) and that the ratio $\hbar\omega/E_0$ is very small, this expression can be rewritten as [7]

$$\lambda_b = \lambda_{b0} \frac{1}{1 - (\frac{v_0}{c})^2(1 - n^2)}. \quad (4)$$

Here $n = kc/\omega$ is the refractive index of the dielectric slab and

$$\lambda_{b0} = 2\lambda_p \left(\frac{E_0}{\hbar\omega} \right) \left(\frac{v_0}{c} \right)^3. \quad (5)$$

It may be assumed that the quantity $E_0 - mc^2 = 50$ keV (the average energy of incident electrons) was sufficiently well fixed in the Schwarz experiments. Therefore, the ratio of the initial electron velocity to the velocity of light in vacuum is $v_0/c = 0.4127$ and $E_0/\hbar\omega = 2.208 \times 10^5$. Then

$$\lambda_{b0} = 1.515 \text{ cm}. \quad (6)$$

In the literature, the following three experimental values for quantity λ_b are presented: 1.70 [2], 1.75 [13], and 1.73 ± 0.01 [14] cm.

The authors of Refs. [2,13,14] did not specify for which of the three above-mentioned dielectric materials these values had been determined. Equation (4) gives the largest value of λ_b for strontium fluoride, $\lambda_b = 1.29$ cm. This material has the smallest value of the refractive index ($n = 1.43$) among the three materials used. As affirmed in Ref. [4], the main material used in the experiments was SiO₂. By using Eq. (4), we obtain $\lambda_b = 1.22$ cm for α -quartz. Thus it appears that the considered quantum-mechanical model does not give the agreement with experiment for λ_b .

The situation, however, can be somewhat improved. As noted in Refs. [6,15], only one propagation mode of the light wave TM₀ can be excited within the slab under the experimental conditions considered. The corresponding wave field can be represented by a superposition of two traveling plane waves, propagating at angles $\pm\alpha$ to the x axis. These waves turn one into another upon total

internal reflection at the slab surfaces. The condition for the appearance of the next mode TM₁ can be written as $d > \lambda_p/2\sqrt{n^2 - 1}$. For α -quartz it means $d > 2040 \text{ \AA}$.

In case the light field is represented by one TM mode, the relativistic quantum-mechanical treatment can be carried out by analogy with the previous case (see also Ref. [15]). Such treatment leads to the same sine term for the stationary spatial modulation as that term in Eq. (1). We obtain the following expression for λ_b :

$$\lambda_b = \lambda_{b0} \frac{1}{1 - (\frac{v_0}{c})^2(1 - n^2 \cos^2 \alpha)}. \quad (7)$$

This formula gives a better value for the spatially beating wavelength, $\lambda_b = 1.47$ cm, for α -quartz if we suppose that the light field within the slab is represented by the TM₀ mode. However, the condition for total internal reflection, $n \cos \alpha > 1$, limits the possibility to improve the agreement between the theory and experiment by using the formula (7). This implies that $\lambda_b = \lambda_{b0} = 1.515$ cm is the upper limit, which cannot be exceeded by any formal optimization of the parameters n and d .

Formally, the values $\lambda_b = 1.70 - 1.75$ cm can be obtained by using formula (7) if we suppose that the dominant role in the effect is played by some radiation mode. In this case the laser light simply crosses the slab. However, the angles between the input laser light and the slab surface must be very large, $53 - 63^\circ$, in confrontation with the described experimental conditions.

The wavelength λ_b arises in the considered quantum-mechanical models as a result of the beat among three plane waves representing free electrons. These waves are characterized by the quantum numbers E_n and \mathbf{p}_n ($n = 0, \pm 1$). The values of E_n and p_{nx} are determined uniquely by the conservation of energy and the x component of quasimomentum in the elementary act of the electron-photon interaction inside the dielectric slab. Then the values of p_{nz} are determined by the relativistic relationship (2). These factors hold for both the one-particle and many-particle considerations.

Thus the quantity λ_b is determined by the simple but fundamental propositions of the physical theory. Therefore, we can conclude that the quantum models that use the electron plane waves ("one-dimensional" in terms of Ref. [16]) have no the chance of resolving the discrepancy of more than 10% between theory and experiment for the quantity λ_b . This statement remains valid even if we take into account some uncertainty of the published experimental data on the parameters n and d .

An attempt to improve the agreement with experiment for λ_b has been made in Ref. [14]. An expression obtained in Ref. [17] was used for a momentum density of a light wave in a refracting medium. The agreement has been obtained at the cost of repudiating the conservation of the x component of quasimomentum in the electron-photon interaction inside the slab. However, such a step is incorrect because the slab length in the x direction can be considered infinite for the conditions

of the Schwarz experiments. At the same time, as noted in Ref. [17], the quasimomentum must be conserved in a uniform medium. Finally, the formal agreement with experiment obtained in Ref. [14] for the case of the plain light wave loses any sense for the light field represented by the waveguide mode TM_0 . Calculation shows that the angle α is sufficiently large ($\alpha = 46^\circ$ for α -quartz).

Another contradiction between the theory and experiment can be added to the ones noted above. The Schwarz experiments definitely indicate [2,16] that there must be the maximum of the Schwarz-Hora radiation intensity at the film surface $z = 0$, i.e., there must be the cosine instead of the first sine in formula (1). This problem was discussed in Ref. [16]. Then the more rigorous treatment by the same authors [11] has in fact confirmed that the theory gives the sine in the dependence of the beating effect on the distance z . This is in accordance also with Ref. [18]. Thus this is one more reliably established discrepancy between the theory and the experiment.

In conclusion, the upper limit $\lambda_b = 1.515$ cm has been obtained for the theoretically permissible values of the spatially beating wavelength for the conditions of the Schwarz experiments. It does not seem possible to account for the large discrepancy between this value and the experimental values ($\lambda_b^{\text{expt}} = 1.70 - 1.75$ cm) on the basis of the existing theoretical models. If we add here the other problems mentioned above (the radiation intensity, the dependence on laser light polarization, and the initial phase of the spatial beating), the situation becomes worse. To clear up the situation, it is desirable to obtain more detailed experimental information, which ought to include, for instance, the dependence of λ_b on the electron velocity v_0 and the refractive index of the dielectric film. Unfortunately, the results of the Schwarz experiments have not been reproduced by other groups up to now. Since 1972 no reports on the results of further attempts to repeat those experiments in other groups have appeared while the failures of the initial such attempts have been explained by Schwarz in Ref. [3].

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